GRAPHIC EQUALIZER DESIGN USING HIGHER-ORDER RECURSIVE FILTERS

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ABSTRACT

A straight-forward design of graphic equalizers with minimum-phase behavior based on recently developed higher-order band-shelving filters is presented. Due to the high filter order, the gain in one band is almost completely independent from the gain in the other bands. Although no special care will be taken to design filters with complementary edges except for a suitable definition of the cut-off frequencies, the resulting amplitude deviation in the transitional region between the bands will be sufficiently low for many applications.

1. INTRODUCTION

Equalizers are common audio effect devices. With their configurable magnitude response, they allow the signal's spectrum to be shaped, to emphasize or deemphasize certain frequency bands or to compensate non-ideal reinforcement equipment or room acoustics. Equalizers allow the specification of desired gains for specified bands. These bands can either be adjustable by the user in so called parametric equalizers, or can be fixed for graphic equalizers, which we will focus on in this paper.

When the gains of the different bands of a graphic equalizer are controlled with sliders, the positions of the sliders' knobs can be understood as samples of the equalizers magnitude response at different frequencies. Hence, the sliders yield a graphic representation of the magnitude response, which is why this form of equalizer is called "graphic".

The bands are usually distributed logarithmic across the frequency, to match human perception. Common designs include octave and $1/3$-octave distribution.

When an equalizer (or any kind of effect device) is used in a live performance, the processing delay is of concern. While the maximum allowable total delay may be a matter of discussion, it is safe to require each individual device to have the lowest possible processing delay, to allow cascading of several devices. This mandates using minimum-phase recursive filters instead of linear-phase FIR filters.

Since a general IIR filter design is computationally intensive, the equalizer is usually implemented as a cascade of sub-filters, each of which realizes the desired gain for one band and ideally has unity gain in the other bands.

In the following, we will first go through the general steps of designing an equalizer out of such sub-filters. While doing so, we will also discuss the properties these sub-filters should ideally have. We will then present a rather straight-forward design for such filters, to finally employ them in two design examples to discuss their specific features.

2. EQUALIZER DESIGN USING SHELVING FILTERS

When designing an equalizer using per-band sub-filters, the main task is to specify the involved frequencies to arrive at a specification for the sub-filters. Let us denote the normalized lower and upper cut-off frequencies of the i-th band with $\Omega_{\mathrm{L},i}$ and $\Omega_{\mathrm{U},i}$, respectively. Furthermore, we define the bandwidth as the difference

$$\Omega_{\mathrm{B},i} = \Omega_{\mathrm{U},i} - \Omega_{\mathrm{L},i} \tag{1}$$

and the center frequency as the geometric mean

$$\Omega_{\mathrm{C},i} = \sqrt{\Omega_{\mathrm{U},i} \cdot \Omega_{\mathrm{L},i}} \tag{2}$$

of the cut-off frequencies. The frequency bands shall be adjacent, that is $\Omega_{\mathrm{U},i} = \Omega_{\mathrm{L},i+1}$. The logarithmic distribution can equivalently be specified using the cut-off frequencies $\Omega_{\mathrm{L},i+1} = R \cdot \Omega_{\mathrm{L},i}$ or center frequencies $\Omega_{\mathrm{C},i+1} = R \cdot \Omega_{\mathrm{C},i}$, from which immediately follows that also $\Omega_{\mathrm{B},i+1} = R \cdot \Omega_{\mathrm{B},i}$. Note that for an octave equalizer $R = 2$ and for a 1/3-octave equalizer $R = \sqrt[3]{2} \approx 1.26$. We shall base our design on the band center frequencies, so that it is convenient to express the other parameters as

$$\Omega_{\mathrm{L},i} = \frac{1}{\sqrt{R}}\, \Omega_{\mathrm{C},i}, \tag{3}$$

$$\Omega_{\mathrm{U},i} = \sqrt{R}\, \Omega_{\mathrm{C},i} \tag{4}$$

and

$$\Omega_{\mathrm{B},i} = \left(\sqrt{R} - \frac{1}{\sqrt{R}}\right) \Omega_{\mathrm{C},i}. \tag{5}$$

Ideally, the sub-filter for the i-th band has magnitude of the desired gain g_i inside the band and unity gain outside that band, that is the ideal band-shelving filter which satisfies

$$\left|H_{\mathrm{ideal},i}(e^{j\Omega})\right| = \begin{cases} g_i & \text{if } \Omega_{\mathrm{L},i} \leq \Omega < \Omega_{\mathrm{U},i} \\ 1 & \text{else.} \end{cases} \tag{6}$$

In practice, the sub-filters will of course have transitional regions around the band edges. In order to have a flat magnitude response in cases where the same gain is used in adjacent bands, we therefore demand the filters to have (at least nearly) complementary band-edges. In particular, we require

$$\left|H_i(e^{j\Omega_{\mathrm{L},i}})\right| = \left|H_i(e^{j\Omega_{\mathrm{U},i}})\right| = \sqrt{g_i}, \tag{7}$$

so that at the cut-off frequency, when $g_i = g_{i+1} = g$, we have

$$\left|H_i(e^{j\Omega_{\mathrm{U},i}}) \cdot H_{i+1}(e^{j\Omega_{\mathrm{U},i}})\right| = g. \tag{8}$$

The filter edges should also be sufficiently steep, so that the inter-band influence is negligible even if two adjacent bands are configured to vastly different gains. To accomplish this, we will use higher-order filters.

3. SHELVING FILTER DESIGN

The design of the individual shelving filters of which the equalizer is to be composed is based on a continuous-time minimum-phase low-shelving prototype derived from a Butterworth low-pass design [1, 2] as given by

$$H_{\mathrm{LS}}(s) = \prod_{m=1}^{M} \frac{s + e^{j\alpha_m} \sqrt[M]{g}}{s + e^{j\alpha_m}}, \quad \alpha_m = \left(\frac{1}{2} - \frac{2m-1}{2M}\right)\pi, \tag{9}$$

where M is the filter order. Note that for $M = 1$ and $M = 2$, this design reduces to the well known first- and second-order low-shelving filters, respectively [3]. The magnitude response of the low-shelving prototype is

$$|H_{\mathrm{LS}}(j\omega)| = \sqrt{\frac{\omega^{2M} + g^2}{\omega^{2M} + 1}}. \tag{10}$$

By choosing $|H_{\mathrm{LS}}(j\omega_{\mathrm{B}})| = \sqrt{g}$, we find that for the normalized prototype of (9), the cut-off frequency $\omega_{\mathrm{B}} = \sqrt[2M]{g}$.

Assuming even filter order M, we can rewrite (9) in terms of real-valued second-order sections as

$$\begin{aligned} H_{\mathrm{LS}}(s) &= \prod_{m=1}^{M/2} \frac{s^2 + 2\cos(\alpha_m)\sqrt[M]{g}s + \sqrt[M]{g}^2}{s^2 + 2\cos(\alpha_m)s + 1} \\ &= \prod_{m=1}^{M/2} 1 + 2V\frac{1 + c_m s}{s^2 + 2c_m s + 1} + V^2 \frac{1}{s^2 + 2c_m s + 1} \end{aligned} \tag{11}$$

with $c_m = \cos(\alpha_m)$ and $V = \sqrt[M]{g} - 1$.

From this, we obtain a digital filter by applying the bilinear transform [4] and frequency-scaling with

$$s = \frac{1}{K} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \tag{12}$$

yielding

$$\begin{aligned} &H_{\mathrm{LS}}(z) = \\ &\prod_{m=1}^{M/2} \left(1 + 2V \frac{K\left(K + c_m + 2Kz^{-1} + (K - c_m)z^{-2}\right)}{1 + 2Kc_m + K^2 + (2K^2 - 2)z^{-1} + (1 - 2Kc_m + K^2)z^{-2}}\right. \\ &\left. + V^2 \frac{K^2\left(1 + 2z^{-1} + z^{-2}\right)}{1 + 2Kc_m + K^2 + (2K^2 - 2)z^{-1} + (1 - 2Kc_m + K^2)z^{-2}}\right). \end{aligned} \tag{13}$$

The frequency-scaling constant K is chosen to map the desired digital cut-off frequency Ω_{B} to the analog cut-off frequency $\omega_{\mathrm{B}} = \sqrt[2M]{g}$, that is

$$K = \frac{1}{\sqrt[2M]{g}} \tan\left(\frac{\Omega_{\mathrm{B}}}{2}\right). \tag{14}$$

Given the low-shelving filter, a band-shelving filter as required for the equalizer of order $2M$ can be derived by a low-pass to band-pass frequency transformation according to

$$H_{\mathrm{BS}}(z) = H_{\mathrm{LS}}\left(z \frac{\cos\Omega_{\mathrm{M}} - z}{1 - \cos\Omega_{\mathrm{M}} z}\right), \tag{15}$$

which shifts the filter such that the maximum (or minimum for $g < 1$) of the transfer function is reached at Ω_{M}, while the bandwidth Ω_{B} and the minimum-phase behavior are retained [5]. In practice, the substitution may be performed by replacing the unit-delays of the low-shelving filter with the all-pass

$$A(z) = z^{-1} \frac{\cos\Omega_{\mathrm{M}} - z^{-1}}{1 - \cos\Omega_{\mathrm{M}} \cdot z^{-1}}. \tag{16}$$

One possible realization of the m-th frequency-shifted second-order section of the band-shelving filter $H_{\mathrm{BS}}(z)$ which aims at minimizing the number of different coefficients is shown in Figure 1, where

$$a_{0,m}^{-1} = \frac{1}{1 + 2Kc_m + K^2}. \tag{17}$$

More details about this design and realization aspects can be found in [1, 2].

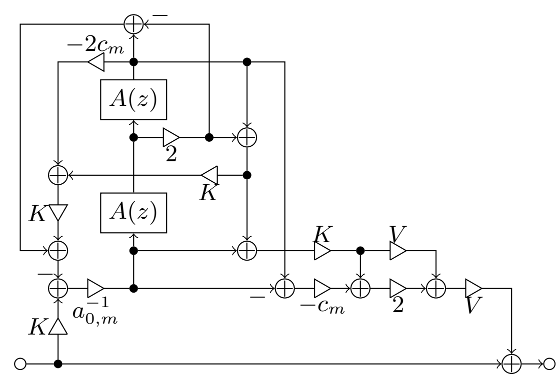

Figure 1: *Realization of the m-th fourth-order (frequency-shifted second-order) section of $H_{BS}(z)$.*

The magnitude response

$$\left|H_{\mathrm{BS}}\left(e^{j\Omega}\right)\right|^2 = \frac{(\cos\Omega_{\mathrm{M}} - \cos\Omega)^{2M} + (K\sin\Omega)^{2M} g^2}{(\cos\Omega_{\mathrm{M}} - \cos\Omega)^{2M} + (K\sin\Omega)^{2M}}, \tag{18}$$

of the filter defined by (15) is depicted in Figure 2 for various maximum-gain frequencies.

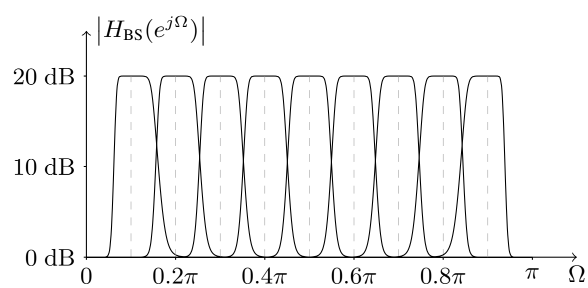

Figure 2: *Magnitude responses of band-shelving filters with order $2M = 12$, gain $g = 10$, bandwidth $\Omega_B = 0.1\pi$ and maximum-gain frequencies $\Omega_M = [0.1\pi, 0.2\pi, \ldots, 0.9\pi]$.*

Comparing these to the required properties stated in section 2, we find that these filters deliver a good approximation of (6), getting even better with higher M of course, as the edges get steeper. To fulfill (7), we have to choose the maximum-gain frequency such that

$$\tan^2\left(\frac{\Omega_{\mathrm{M},i}}{2}\right) = \tan\left(\frac{\Omega_{\mathrm{U},i}}{2}\right) \cdot \tan\left(\frac{\Omega_{\mathrm{L},i}}{2}\right). \tag{19}$$

i	$f_{C,i}$/Hz	$f_{L,i}$/Hz	$f_{U,i}$/Hz	$f_{M,i}$/Hz	$\cos\left(\frac{2\pi f_{M,i}}{f_S}\right)$
1	30	21	42	30	0.999992
2	60	42	85	60	0.999969
3	120	85	170	120	0.999877
4	240	170	339	240	0.999507
5	480	339	679	480	0.998026
6	960	679	1358	960	0.992110
7	1920	1358	2715	1923	0.968500
8	3840	2715	5431	3861	0.874993
9	7680	5431	10861	7862	0.515600
10	15360	10861	21722	17955	−0.702955

Table 1: *Center frequencies $f_{C,i}$, lower cut-off frequencies $f_{L,i}$, upper cut-off frequencies $f_{U,i}$, maximum-gain frequencies $f_{M,i}$ and resulting frequency-shifting coefficients* $\cos(\Omega_{M,i})$ *of the octave equalizer.*

i	K_i	i	K_i	i	K_i
1	0.001168	5	0.018694	9	0.312322
2	0.003300	6	0.052838	10	1.023332
3	0.004673	7	0.074962		
4	0.013201	8	0.213467		

Table 2: *Bandwidth coefficients K_i of the octave equalizer with gains alternating between* 12 dB *and* −12 dB.

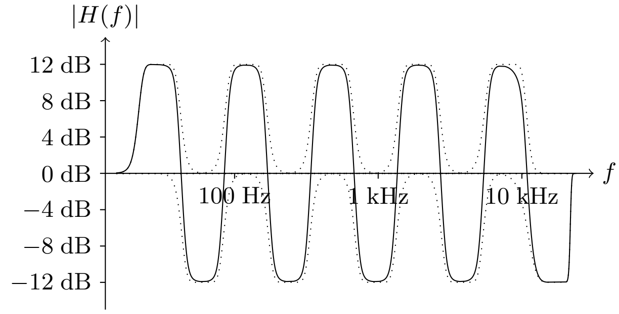

Figure 3: *Magnitude response of the octave equalizer with eighth-order (M=4) band-shelving filters with gain alternating between* 12 dB *and* −12 dB *(solid line) and of the individual filters (dotted).*

i	K_i	i	K_i	i	K_i
1	0.001168	5	0.018694	9	0.312322
2	0.002336	6	0.037407	10	0.724464
3	0.004673	7	0.074962		
4	0.009346	8	0.151123		

Table 3: *Bandwidth coefficients K_i of the octave equalizer with equal gain (*12 dB*) in all bands.*

This is different from known approaches with bi-quad peak filters, where usually $\Omega_M = \Omega_C$. Considering the flatness of the magnitude response in the respective band, it seems however valid to have slightly different Ω_M and Ω_C.

4. DESIGN EXAMPLES

4.1. Octave equalizer

Let us consider a 10-band octave equalizer with center frequencies

$$f_{C,i} = [30\text{ Hz}, 60\text{ Hz}, \ldots, 15360\text{ Hz}]$$

at a sampling frequency $f_S = 48$ kHz. The resulting lower and upper cut-off frequencies $f_{L,i}$ and $f_{U,i}$ from (3) and (4), as well as the frequencies of maximum gain $f_{M,i}$ from (19) and the resulting coefficients $\cos(\Omega_{M,i})$ of the frequency-shifting all-pass are listed in Table 1.

We use a band-shelving filter of eighth-order (i.e., $M = 4$) for each band. For the resulting two fourth-order sections for each band, $c_0 = \cos(3\pi/8)$ and $c_1 = \cos(5\pi/8)$, respectively.

As a first test case, the equalizer is configured to gains alternating between 12 dB and −12 dB, so that $g_i = 3.98107$ and hence $V_i = 0.412538$ for odd i and $g_i = 0.251189$ and hence $V_i = -0.292054$ for even i. Application of (14) yields the K_i of Table 2. The resulting magnitude response in Figure 3 shows that the inter-band influence is sufficiently small, so that the difference between actual and desired gain in each band is hardly perceptible.

To study the imperfections due to non-complementary filter edges, we now set equal gain of 12 dB in all bands, yielding the same $g_i = 3.98107$ and hence $V_i = 0.412538$ for all bands. By application of (14), we find the K_i of Table 3.

The resulting amplitude response of the equalizer and the individual band-shelving filters is depicted in Figure 4, the deviation from 12 dB in Figure 5. The gain is almost exactly 12 dB at the center frequencies and at the cut-off frequencies, so it is not necessary to consider the gains configured in other bands when choosing the g_i for one filter, contrary to other sophisticated equalizer design methods [6]. The influence of one filter in other bands is limited to a small transitional region around the cut-off frequencies, where the magnitude response exhibits slight ripple no higher than that of traditional constant-Q equalizers [7].

Only in the transitional region between ninth and tenth band, the deviation from 12 dB exceeds 1 dB. This is an artifact of the bilinear transform, which results in filters with an upper edge close to the Nyquist frequency to have relatively flat lower edges. Thus, the lower edge of the tenth band and the upper edge of the ninth band have significantly different slopes, resulting in the larger ripple. For applications where this is not acceptable, the situation could be remedied to some extent by choosing a lower upper edge frequency for the tenth band, which for the current design lies outside the audible spectrum anyway. For example, by arbitrarily setting $f_{U,10} = 18.5$ kHz, the ripple of Figure 6 can be obtained.

4.2. 1/3-Octave equalizer

Next we examine a 30-band 1/3-octave equalizer with center frequencies

$$f_{C,i} = [25\text{ Hz}, 31\text{ Hz}, \ldots, 20319\text{ Hz}],$$

again at a sampling frequency of 48 kHz. As for the octave equalizer, we use eighth-order (M=4) band-shelving filters.

The magnitude response for the same gain of 12 dB in all bands is depicted in Figure 7, the deviation from 12 dB in Figure 8. Again, the ripple stays below 1 dB up to 10 kHz and grows toward the highest bands. By applying the same trick as for the octave equalizer, reducing the upper edge of the highest band from 22807 Hz to 20.5 kHz, the ripple can be reduced to that shown in Figure 9.

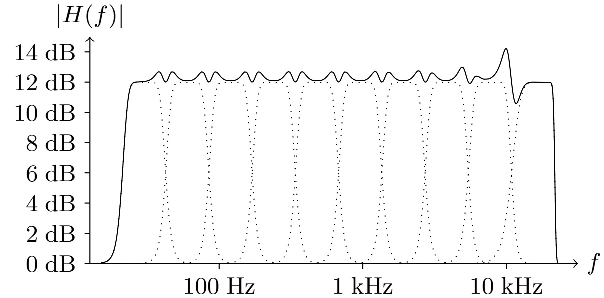

Figure 4: *Magnitude response of the octave equalizer with eighth-order (M=4) band-shelving filters with equal gain (*12 dB*) in all bands (solid line) and of the individual filters (dotted).*

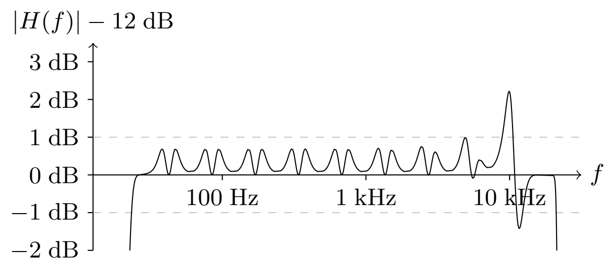

Figure 5: *Deviation of the magnitude response of Figure 4 from the ideal constant* 12 dB.

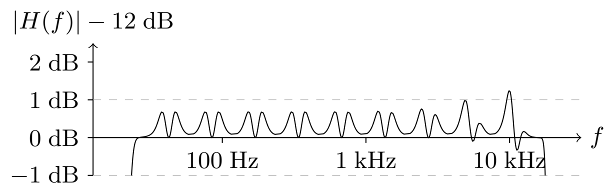

Figure 6: *Deviation of the magnitude response from* 12 dB *for the modified octave equalizer with* $f_{U,10} = 18.5$ kHz.

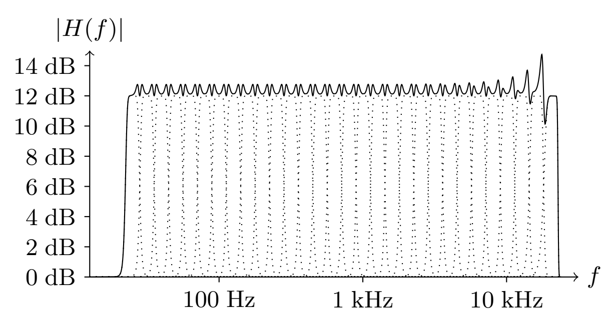

Figure 7: *Magnitude response of the* $1/3$*-octave equalizer with eighth-order (M=4) band-shelving filters with equal gain (*12 dB*) in all bands (solid line) and of the individual filters (dotted).*

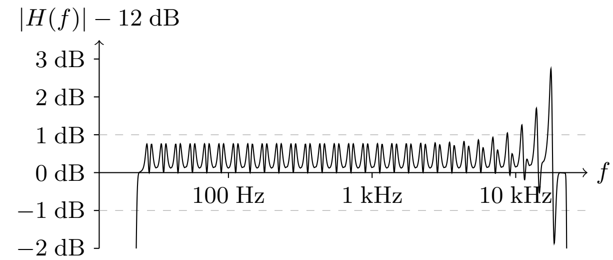

Figure 8: *Deviation of the magnitude response of Figure 7 from the ideal constant* 12 dB.

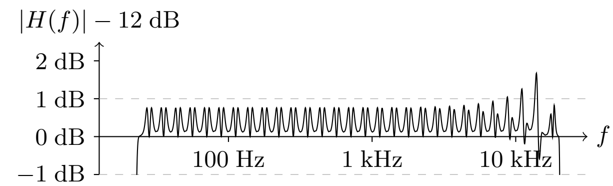

Figure 9: *Deviation of the magnitude response from* 12 dB *for the modified* $1/3$*-octave equalizer with* $f_{U,30} = 20.5$ kHz.

5. CONCLUSIONS

We have presented a straight-forward design of graphic equalizers with minimum-phase behavior based on higher-order band-shelving filters. Thanks to the high filter order, the inter-band influence is very small, that is the gain in one band is almost completely independent from the gain in the other bands. Although no special care has been taken to design filters with complementary edges except for a suitable definition of the cut-off frequencies, the resulting amplitude deviation in the transitional region between the bands is very low. Despite a slight increase at high frequencies, the amplitude ripple should be sufficiently low for most applications.

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